MATH 119: Quiz 3

Name: key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. Suppose

$$f(x) = x(x-1) \qquad g(x) = x+3$$

Evaluate the following and expand/combine like terms:

(a) $f \circ f$

$$= f(f(x)) = f\left(x(x-1)\right) = x(x-1)\left(x(x-1)-1\right)$$

$$= \underbrace{x(x-1)}_{a}(x^2-x-1) = (x^2-x)x^2 - (x^2-x)x - (x^2-x)$$

$$= x^4 - x^3 - x^3 + x^2 - x^2 + x$$

$$= \boxed{x^4 - 2x^3 + x}$$

(b) $f \circ g$

$$= f(g(x)) = f(x+3) = (x+3)\left((x+3)-1\right)$$

$$= \underbrace{(x+3)}_{a}(x+2) = (x+3)\cdot x + (x+3)\cdot 2 = x^2 + 3x + 2x + 6$$

$$= \boxed{x^2 + 5x + 6}$$

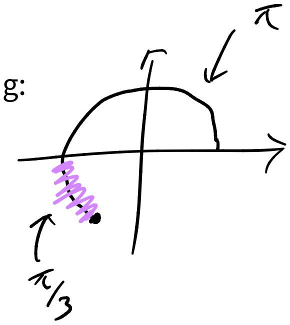
(c) $f(g(0))$

$$= f(0+3) = f(3) = 3\cdot(3-1)$$

$$= 3 \cdot 2 = \boxed{6}$$

2. Evaluate the following:

(a) $\sin\left(\frac{4\pi}{3}\right)$

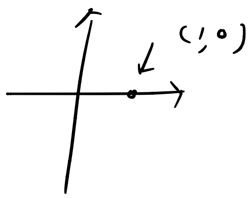


① $\bar{t} = \frac{\pi}{3}$

② sin is negative in III

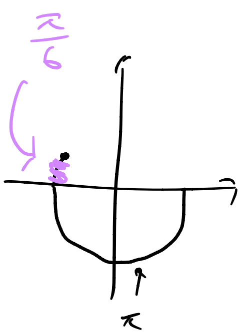
$$\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = \boxed{-\frac{\sqrt{3}}{2}}$$

(b) $\cos(0)$



$$\cos(0) = \boxed{1}$$

(c) $\tan\left(-\frac{7\pi}{6}\right)$



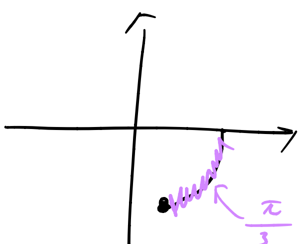
① $\bar{t} = \frac{\pi}{6}$

② tan is negative in II

$$\tan\left(-\frac{7\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$$

$$= \boxed{-\frac{1}{\sqrt{3}} \\ = -\frac{\sqrt{3}}{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

(d) $\cos\left(\frac{-\pi}{3}\right)$



① $\bar{t} = \frac{\pi}{3}$

② cos is positive in IV

$$\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$$

3. The equation of the unit circle is $x^2 + y^2 = 1$. Why is the Pythagorean identity

$$\sin^2 x + \cos^2 x = 1$$

true?

The identity can be written as

$$\sin^2 t + \cos^2 t = 1$$

Any t value results in a terminal point $P(x, y)$. Since terminal points lie on the unit circle and $\sin(t) = y$, $\cos(t) = x$, substituting x for $\cos(t)$ and y for $\sin(t)$ in the equation of the unit circle gives the identity.